Multiresolution analysis with GRASS

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Abstract

The ability of gerarchically manage data available in a Geographical Information System is becoming increasingly important, given the huge data quantity provided by the modern surveying techniques. In particular, the spatial features of an element must be framed in an intrensically multiresolution structure with an efficient management of the information at different resolutions. Four new modules have been created for the multiresolution analysis with GRASS. These modules use the discrete wavelet transform to perform a MultiResolution Analysis (MRA) by the Mallat algorithm, which has been extended for the discrete bidimensional case. Same applications exploiting the potentiality of the technique have been carried out, among these the filtering of LIDAR data and the automatich shape recognition.

1. Introduction

The ever growing data size available for the use in a Geographical Information System poses a threat on the real usability of such data, since the increasing data elaboration capability of GISs can not keep up with this exponential growth.

It is therefore advisable, if not necessary, the development of a hierarchical approach to the management and the elaboration of geographical data. This would result in a multiresolution, both in the geometric and semantic senses, approach to the threatment of geographical information for an efficient management at each resolution level. This work represents a first step for the solution the geometrical side of this problem. This is accomplished by MultiResolution Analysis (MRA) performed using the wavelets bases in the Mallat algorithm, extended for the finite bidimensional case and therefore suitable for the raster maps of a GIS.

2. Signal representation

Signal representation is a frequent issue in GISs, even if most of the times this is hidden by the system from the final user. The signal representation implies the choice of a base for the space where the signal lies: the actual data stored and elaborated by the system are the coefficients that describes the signal with respect to the chosen base. The choice of the base is fundamental for the way data are managed, for the effectiveness of operations as well as for their efficiency. The selection of the base must be made so that a *good* representation of the signal is possible and good analitycal properties are provided, such as smoothness, symmetry, fast decay and localization. In particular, the localization property, expressed by the fact that only a few coefficients are modified by a signal variation in one point, is important both numerically, because good data compression rates can be achieved, and for the data analysis, since local features are kept in the signal representation.

The most popular traditional techniques for signal representation are Fourier series and spline. The Fourier technique writes the function describing the signal as a sum of armonic functions, thus achieving the localization property in frequency. However, the localization in space (or time) is lost: a variation of the signal in one sample point modifies all the series' coefficients. Splines use stepwise polinomial functions, providing localisation in space, but a variation of the value of a sample point propagates its effects on the whole frequency range, therefore there is not localization in frequency. In general, it is not possible to achieve arbitrary localization both in space and frequency, as stated by the Heisemberg uncertainty principle. However, it is possible to substitute *frequency* with *scale* and to provide bases which are local both in space (or time) **and** in scale. There exists a way of providing a signal representation which is local both in space and in scale by using the wavelet bases.

3. Wavelets and MultiResolution Analysis

Wavelets are functions generated by dilation and traslation of a single function ψ called the *mother function*:

$$\psi_{m,n}(t) = 2^{-m/2}\psi\left(2^{-m}t - n\right) \tag{1}$$

The definition of the spaces whose wavelets represent a base and a "recipe" for the explicit determination of the ψ function can be conveniently carried out in the context of Multiresolution Analysis.

MultiResolution Analysis regards a function $f \in L^2(R)$ as the limit of approximating functions given by the projection of f on spaces of increasing resolution:

$$f = \lim_{m \to -\infty} P_m f \tag{2}$$

where $P_m f$ with $m \in Z$ gives a *smooth version* of f, higher m results in a larger smoothing effect.

A MultiResolution Analysis (MRA) consists in a ladder of nested spaces $V_m \subset L^2(R)$ with $m \in Z$

$$\cdots \subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \subset \ldots \tag{3}$$

so that

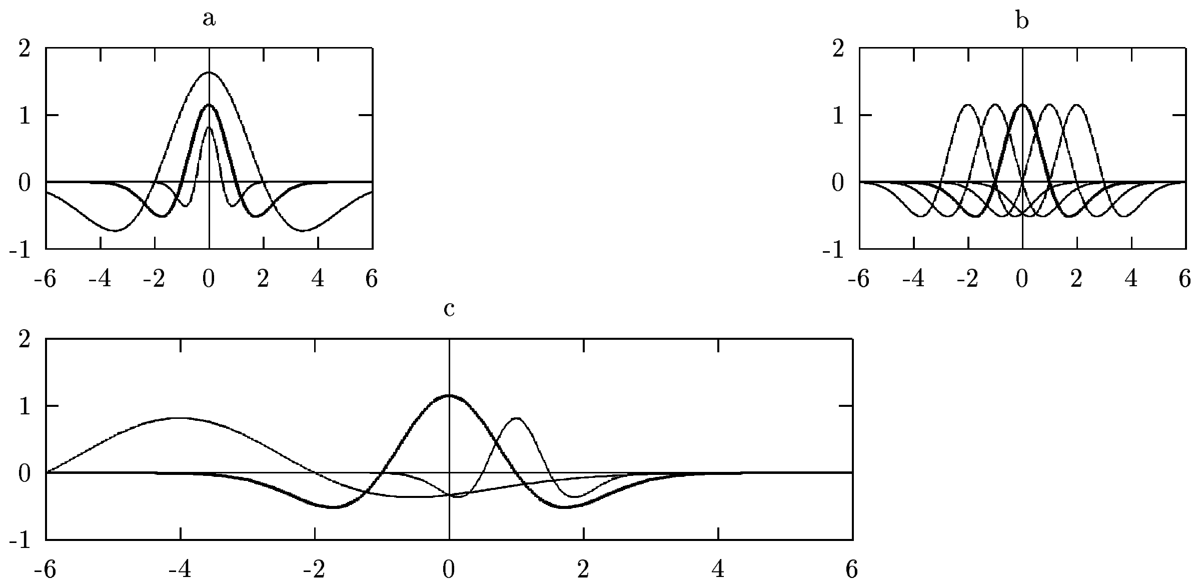

Figure 1: Waveletes family generated by the "Mexican Hat" mother wavelet (thicker) with dilation ratios of 0.5, 1 and 2 (a); with integer translation by -2 and +2 (b); with translation and dilation of a factor 2 (right) and with translation and dilation of factors -2 e 0.5 (left) (c).

$$\bigcap_{m \in Z} V_m = \{0\} \tag{4}$$

$$\overline{\bigcup_{m \in Z} V_m} = L^2(R) \tag{5}$$

and

$$f \in V_m \Leftrightarrow f(2\cdot) \in V_{m-1} \tag{6}$$

i.e.

$$f \in V_m \Leftrightarrow f(2^m \cdot) \in V_0 \tag{7}$$

Moreover, there must exist a function ϕ, so called *scaling function* or *father function* $\phi \in V_0$ so that it generates a family of functions $\phi_{m,n}$

$$\phi_{m,n}(t) = 2^{-m/2}\phi\left(2^{-m}t - n\right) \tag{8}$$

which constitutes a base of V_m, i.e. $V_m = \overline{\text{linear span}\{\phi_{m,n},\ n \in Z\}}$. Equation (6) ensures that the different V_m ccorrespond to different scales; since $\{\phi_{m,n}\}$ is a base of V_m the condition of tralation invariance of f with respect to its belonging to V_m holds:

$$f \in V_m \to f(\cdot - 2^m n) \in V_m \;\; \forall n \in Z \tag{9}$$

Working with discrete data, as it happens with real signals and thus with data in a GIS, only the following nested spaces must be taken into account

$$\cdots \subset V_2 \subset V_1 \subset V_0 \tag{10}$$

where the space V_0 has the maximum resolution.

M.R.A. leads to the construction of a wavelets base through the definition of the spaces W, representing the "difference" between two consecutive spaces V; the W_m space is the orthogonal complement of W_m in V_{m-1}:

$$V_{m-1} = V_m \oplus W_m \tag{11}$$

and

$$W_m \perp V_m \tag{12}$$

i.e.

$$W_m = (P_{m-1} - P_m)\, L^2(R) \tag{13}$$

A space V_m can be written as, for $m < M$,

$$V_m = V_M \oplus \bigoplus_{k=0}^{M-m-1} W_{M-k} \tag{14}$$

Therefore the W_m are orthogonal spaces and sum up to $L^2(R)$

$$L^2(R) = \bigoplus_{m \in Z} W_m \tag{15}$$

The W_m spaces inherit from the V_m spaces the propriety (6):

$$f \in W_m \Leftrightarrow f(2^m \cdot) \in W_0 \tag{16}$$

It is finally possible to find a function ψ so that the $\psi_{m,n}$, expressed by

$$\psi_{m,n}(t) = 2^{-m/2}\psi\left(2^{-m}t - n\right) \tag{17}$$

are an orthonormal base for W_m. The coefficients of a wavelets transform $\langle f, \psi_{m,n} \rangle$ correspond to the difference between two subsequent approximation of the f, $P_{m-1}f$ e $P_m f$. Because of (15) and (16), $\{\psi_{m,n},\ m, n \in Z\}$ is an orthonormal base of $L^2(R)$, while because of (16) if $\{\psi_{0,n},\ n \in Z\}$ is an orthonormal base of W_0 then $\{\psi_{m,n},\ n \in Z\}$ is an orthonormal base of W_m.

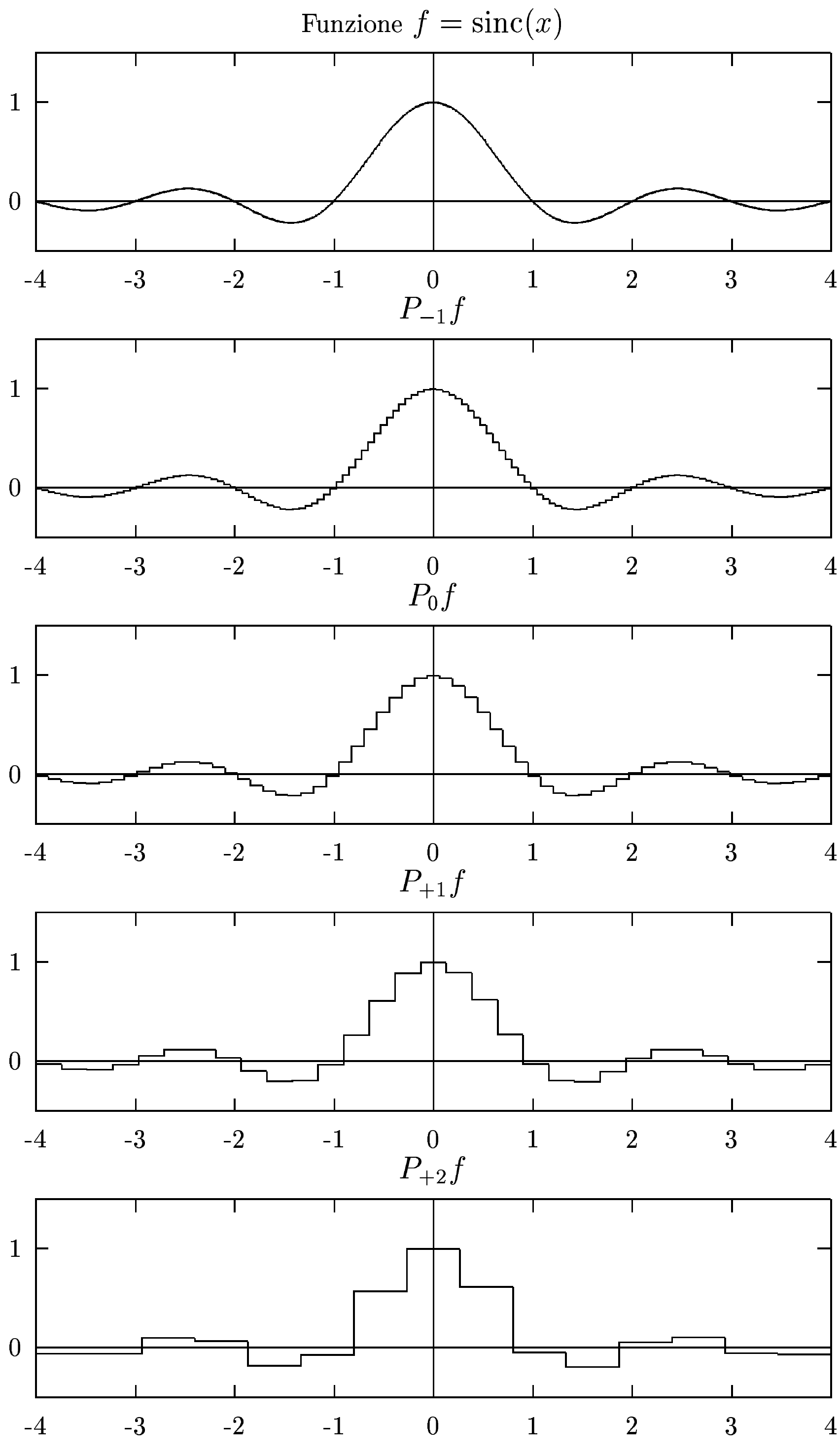

Figure 2: MultiResolution Analysis of the $\mathrm{sinc}(x)$ function.

4. Exension to finite bidimensional domains and base choice

The application of a theory such as that underlying MRA requires some additions to take into account the fact tha real world signals are discrete, finite and over finite (1D or 2D) domains. The deployment of the wavelets in the case of discrete signals can be carried out easily as it is relatively easy to code a program for the discrete wavelets transform (DWT) for analysis and synthesis. Infact, the application of DWT to discrete signal consists in the recursive application of sums over the coefficients of the discrete signal. The recursivity of the algorithms makes the coding of the DWT very efficient.

The application of DWT to real, and therefore finite, signals force to find a way to deal with frontiers, where the lack of data beyond the data stream limit must be compensated to avoid distorsions (errors) on the nearby coefficients. This is a classical problem also for other spectral data analysis techniques such as Fourier transform. There exist several approaches which are based on the filling of the empty coefficients beyond the frontier with *suitable* values or on the use of specific bases for the transform of coefficients near the edges.

The following approaches are the most common:

zero padding the data vector is extended by adding null values beyond the edges, with a number of entries that allows the determination of the value on the edge;

periodicization data are extended by periodicizing the signal, i.e. by repeating the data sequence beyond the edge, the first value outside the right frontier is the first in the data sequence i.e. the coefficient at the other edge;

reflexion data is "reflexed" thru the edge, the first value outside the frontier is the first coefficient inside the edge and so on;

use of a custom base at the edge this technique uses a modified base near the edge so that the coefficient used for the calculation of the DWT od the values near the frontier do not "spill out" the actual data vector. This has the drawback of giving up (a small) computational efficiency.

There are different ways of building a wavelets base for a 2D space:

tensorial product of two bases on the two axes. This is the easiest and most intuitive way, but it has the major drawback of combining all the scales and positions, it is therefore impossibile to separate the contribution at different scales for the two components.

non separable 2D bases built as a product of two 1D functions using an equation such as

$$\psi_{m,n}(\mathbf{x}) = 2^{-m/2}\psi(A\mathbf{x} - n) \tag{18}$$

where the dilation fator is represented by the matrix A.

tensorial product of two (1D) spaces in a MRA framework. The wavelets base of the 2D space has a number of position indexes equal to the number of variables but only one scale index, therefore no mixing between different scales in the components occours.

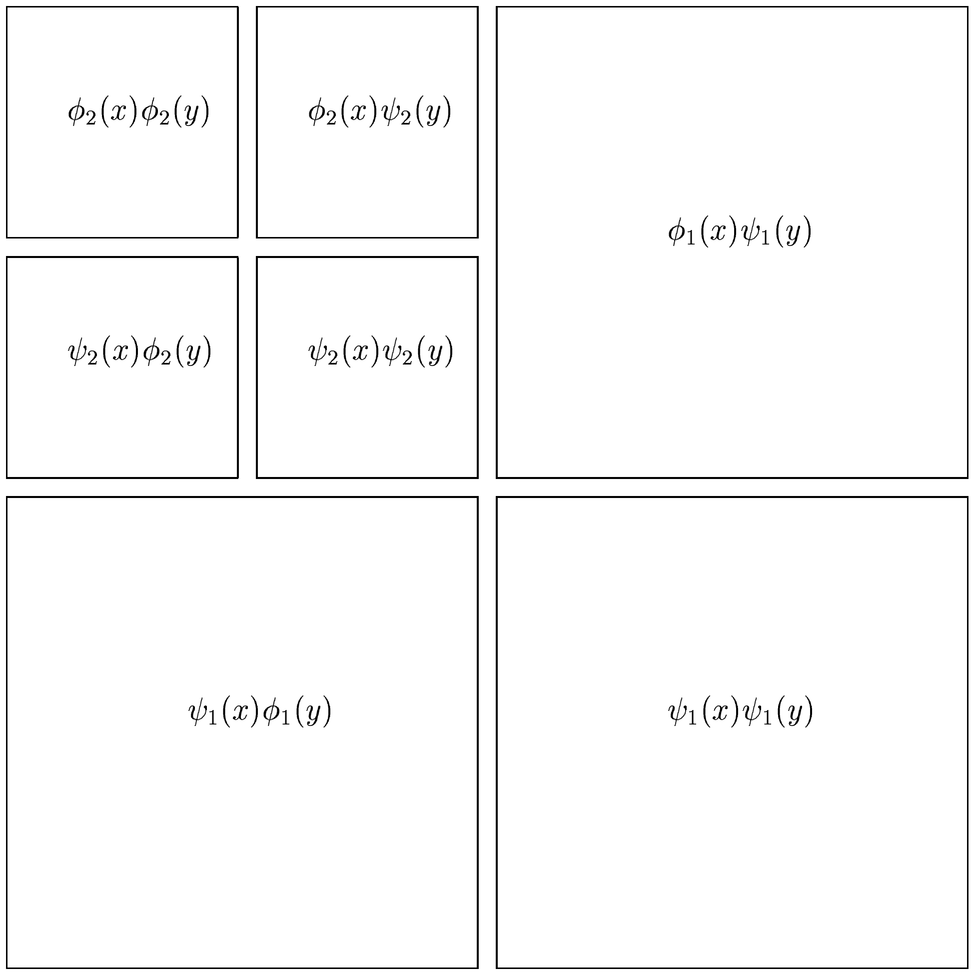

Figure 3: 2D signal analysis using wavelets bases built from the product of two 1D MRA. The bases for the decomposition of the spaces $\mathbf{V}_m$ for $m = 1, 2$ are shown.

The last approach is that most commonly used because of its ability to keep separate the contributions at different scales. The resulting partition of a 2D space is shown in figure 3: the upper left corner contains the signal at lower resolution and it is decomposed in a further step (the figure shows the bases after two decomposition steps).

The choice of the wavelets base depends on the features of the signal to be analysed and on the type of analysis. In general the following features are required:

1. smoothness and symmetry;
2. fast vanishing;
3. maximum possible number of vanishing moments.

These features grant that the signal is transformed by DWT with an high number of null coefficients while retaining a good approximation of the original (discrete) function. Unfortunately, these requests are contradictory and therefore a choice must be made according to the analysis purpose.

The only way to have orthogonal real value symmetric bases is to use different bases for analysis and synthesis, the so-called *biothogonal* bases.

5. Algorithm

The application and implementation of a DWT is usually done by the Mallat Algorithm, which implements directly the structure of the MRA. The approach is similar to that used for the construction of the laplacian pyramids.

The original (discrete) signal is split into two data sequences each half long the original sequence. The first sequence contains a smoothed version of the signal at lower resolution, while the second sequence carries the differences between the original signal and the smoothed version.

The original function is written with respect to a base of the V_0 space, i.e. with respect to the base given by the *father function*

$$f = \sum_n c_n^0 \phi_{0,n} \tag{19}$$

and then

$$f(x) = \sum_n c_n^0 \phi(x-n) \tag{20}$$

where $f \in V_0$. It is possible to define two projectors: P_m projecting f into the space V_m with resolution m and Q_m projecting f into the space W_m with resolution m, which holds the differences between the two resolution levels. The function f is split in two using the fact that $V_0 = V_1 \oplus W_1$:

$$f = P_1 f + Q_1 f \tag{21}$$

and, expressing explicitly the spaces' bases

$$P_1 f = \sum_k c_k^1 \phi_{1,k} \tag{22}$$

$$Q_1 f = \sum_k d_k^1 \psi_{1,k} \tag{23}$$

The c^1 sequence represents a smoothed version of the original sequence, while the d^1 sequence bear the differences between c^0 and c^1. The key point is the possibility of writing the c^1 and d^1, and therefore the c^m and d^m sequences for each m space, as functions of the sole c^{m-1} sequence at the previous step. It is possible to define two discrete filters $h(n)$ and $g(n)$ so that c^m and d^m are expressed as

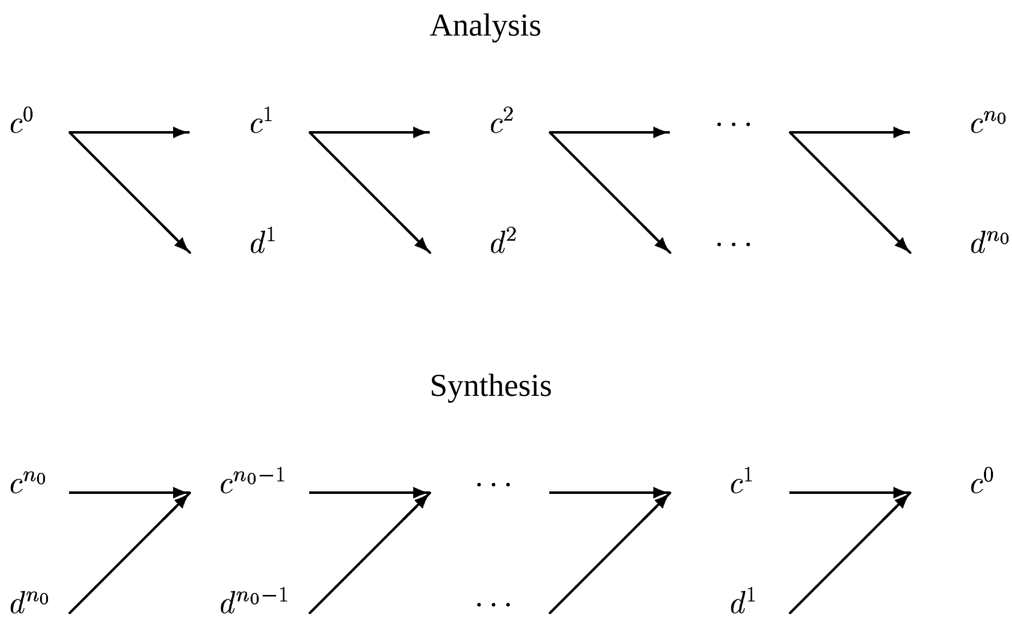

Figure 4: Tree scheme of the Mallat's algorithm for 1D signals.

$$c_k^m = \sum_n h_{n-2k} c_n^{m-1} \tag{24}$$

$$d_k^m = \sum_n g_{n-2k} c_n^{m-1} \tag{25}$$

The filter coefficients $h(n)$ and $g(n)$ depend on the chosen MRA i.e. on the wavelets base in use. For these coefficients a normalisation condition holds

$$\sum_n h_n = \sqrt{2} \tag{26}$$

and

$$\sum_n g_n = 0 \tag{27}$$

The wavelet base involved in the MRA must take into account the fact that the domain is finite, as discussed in paragraph 4., otherwise there exist some non zero coefficients that "spill out" the boundaries of the data packets: negleting them would lead to a distortion on the analysis and all subsequent operations on the data.

The wavelets analysis and synthesis of a 2D signal are performed by the tensor product of two 1D analysis. The original, 2D, sequence c_0 is split onto four sequences corresponding the lower resolution signal, the differences in the horizontal, in the vertical and in the diagonal direction (fig. 8).

The algorithm stops when only one coefficient represting the lower resolution signal remains.

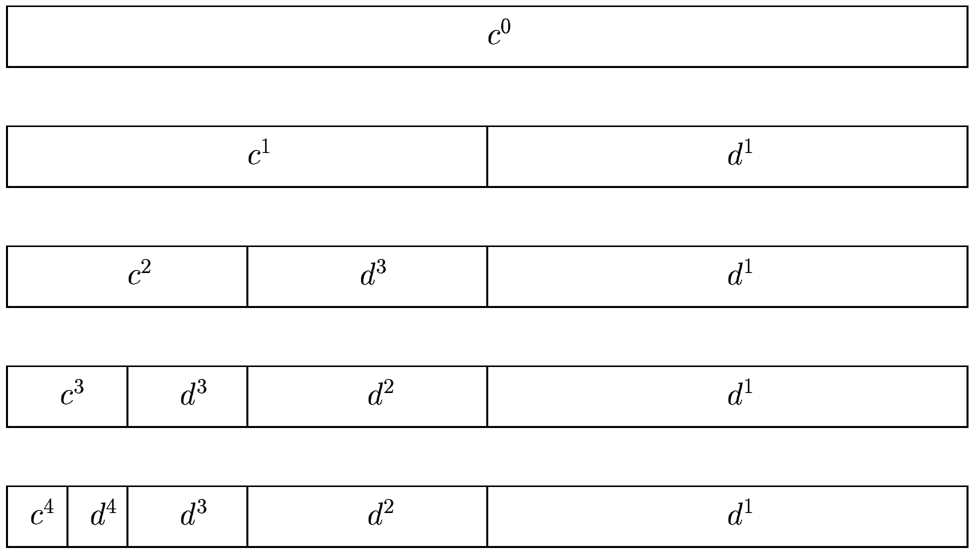

Figure 5: Scheme of the Mallat's algorithm for DWT of a 1D signal: the wavelets d coefficients and the scaling functions c coefficients at each analysis level are shown.

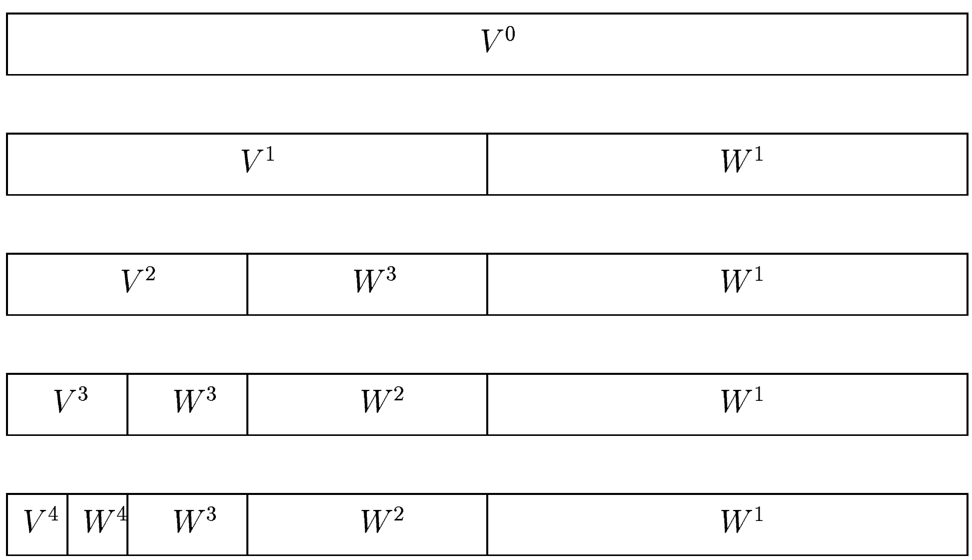

Figure 6: Scheme of the Mallat's algorithm for DWT of a 1D signal: the spaces of the MRA are shown.

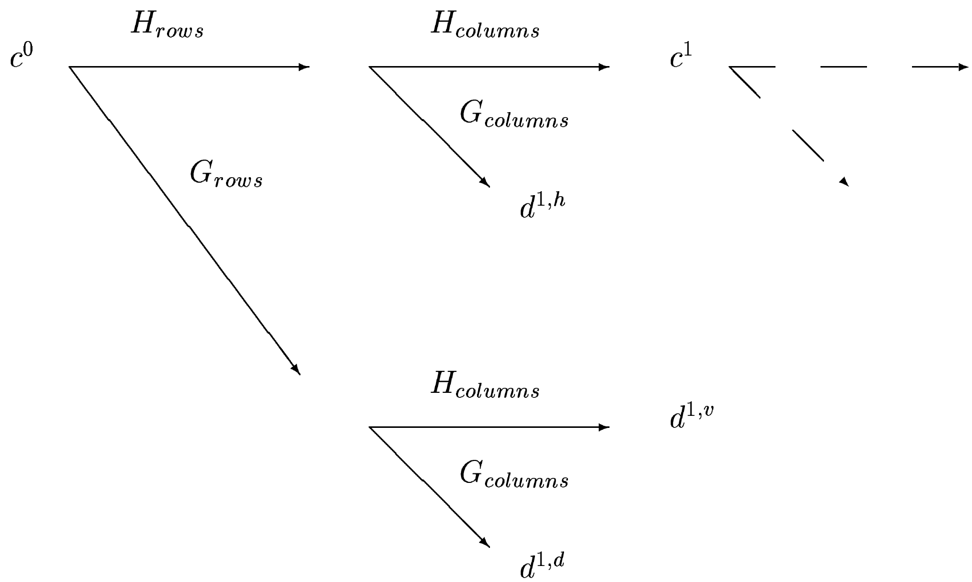

Figure 7: Tree scheme of the Mallat's algorithm for 2D signals.

6. MRA and GRASS

The MultiResolution approach in a GIS can provide significant advantages both as a way of representing the data and as a way to efficently perform operations on them. Infact, most of the geographical data can be represented and analysed at different resolutions, depending on the application.

With respect to this MRA allows efficient ways of represent and conditionally select data at different levels of resolution, with each level complitely separated (due to the othogonality of the spaces), as well as effective method of implementing operations on data.

GRASS GIS is a very good choice for the implementation of MRA because of its Open Source development model and because it provides high quality raster data and manipulation functions support. Four new modules for the analysis and synthesis of raster data have been written by Andrea Antonello in collaboration with the author of the present paper. These modules are:

- r.owave.dec
- r.owave.rec
- r.biowave.dec
- r.biowave.rec

While the first two modules perform wavelets analysis and synthesis using orthogonal bases, the latter two use biorthogonal bases, as discussed in par 4.. For a full description of the modules' options

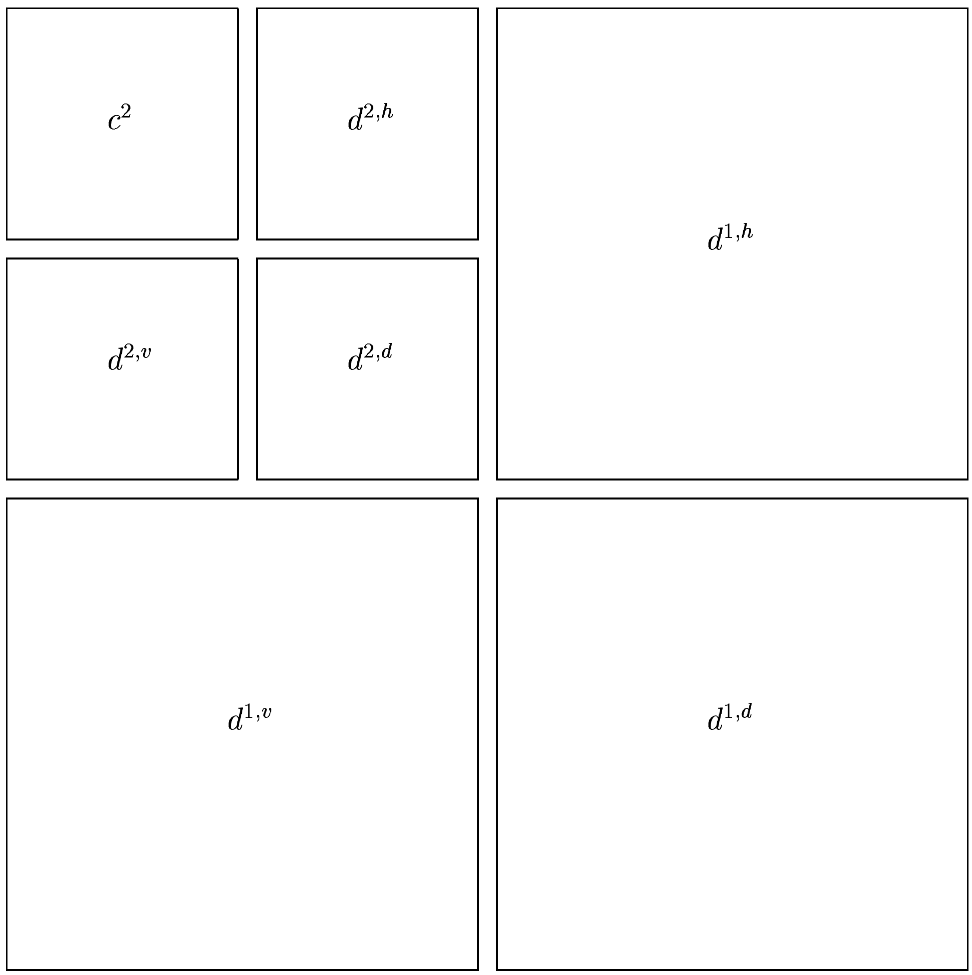

Figure 8: Sub-images (sequences) generated of the Mallat's algorithm for 2D signals.

see [ZA02], for each module the choice of the wavelet base is done by selecting the corresponding filter (as defined by a set of coefficients).

"**r.owave.dec**" computes the orthogonal wavelet transform of a raster map.

Synopsis:
r.owave.dec input=*Image* output=*WavTrans* filter1=*ImpulseResponse*
filter2=*EdgeRi* filterpath=*filterpath*
[*FilterNorm*=n] [*NumRec*=r] [*Haar*=h] [*Edge*=e]

Description:
r.owave.dec computes the two-dimensional discrete wavelet transform (*WavTrans*) of the floating point raster stored in the file *Image*, according to the pyramidal algorithm of Mallat. The coefficients of the filter's impulse response are read from the file *ImpulseResponse*. The coefficients of the the filter's impulse response for special edge processing are read from the file *EdgeRi*. The filters are stored in the folder *filterpath*

Output:
The module gives as output (WavTrans) four images for every processed level, N_output_11 representing the raster data at coarser resolution, N_output_12, N_output_21 and N_output_22 the detail data. N stands for the level of decomposition, output is the name given to the file during runtime as input parameter.

Notes:

- in addition to the input parameters above, during runtime the program asks a path where the file with the information necessary for reconstruction (with r.owave.rec) is stored.

" **r.owave.rec**" reconstructs a raster map from an orthogonal wavelet transform

Synopsis:
r.owave.rec

Description:
r.owave.rec reconstructs a raster map from a sequence of sub images forming a wavelet decomposition, according to the pyramidal algorithm of S. Mallat. The raster map is written into the file RecompImage. *WavTrans* is the prefix name of a sequence of files containing the coefficients of a wavelet decomposition. The coefficients of the filter's impulse response are read from the file *ImpulseResponse*. The coefficients of the filter's impulse response for computing the edge coefficients are read from the file *EdgeIR*.

Output:
The module gives as output the reconstructed raster map called output_reconstructed, where output is the filename provided during runtime as input parameter.

Notes:

- the program runs without command line parameters, the interactive use is mandatory;
- during runtime the first thing the program asks for is whether the auxiliary file produced by r.owave.dec is available or if these informations have to be supplied manually. In the first case only the name and the path of the auxiliary file are required.

" **r.biowave.dec** " computes the biorthogonal wavelet transform of a raster map.

Synopsis:
r.biowave.dec input=*Image* output=*WavTrans* filter1=*ImpulseResponse1*
filter2=*ImpulseResponse2* filterpath=*filterpath*
[*FilterNorm*=n] [*NumRec*=r] [*Haar*=h] [*Edge*=e]

Description:
r.biowave.dec computes the two-dimensional discrete wavelet transform of the floating point raster map stored in the file *Image*, according to the pyramidal algorithm of Mallat, using filter banks (*ImpulseResponse1*, *ImpulseResponse2*) associated to biorthogonal bases of wavelets.

Output:
The module gives as output four raster maps for every processed level, N_output_11 representing the raster map at coarser resolution, N_output_12, N_output_21 and N_output_22 the detail raster maps. N is for the level of decomposition and output is the name given to the file during runtime as input parameter.

Notes:

- in addition to the input parameters above, the program asks runtime a path where to store the file with the information used for reconstruction (with r.biowave.rec);

" **r.biowave.rec**" reconstructs a raster map from a biorthogonal wavelet transform.

Synopsis:
r.biowave.rec

Description:
r.biowave.rec reconstructs a raster map from a sequence of subimages forming a wavelet decomposition, using filter banks associated to biorthogonal bases of wavelets. The image is stored in the file *RecompImage*. *WavTrans* is the prefix name of a sequence of files containing the coefficients of a wavelet decomposition. The coefficients of the filter's impulse responses are read from the file *ImpulseResponse1* and *ImpulseResponse2*.

Output:
The module gives as output the reconstructed image called output_reconstructed, where output is the name indicated during runtime as input parameter.

Notes:

- the program runs without command line parameters, the interactive use is mandatory;
- during runtime the first thing the program asks for is whether the auxiliary file produced by r.biowave.dec is available or if these informations have to be supplied manually. In the first case only name and path of the auxiliary file are required.

These modules have been written by Andrea Atnonello and they are based on the MegaWave library project (http://www.cmla.ens-cachan.fr/Cmla/MegaWave). They are released under the GNU library.

7. Applications

A wide range of applications in a GIS enviroment can benefit of MRA, both for efficiency improvement and for a better data insight, due to the multiresolution representation. In particular the following fields can take advantage of MRA:

efficient representation of signals: this allows effective computations and high data compression;

application of operators in the W spaces: leads to computational efficency;

signal filtering : it is possible to filter out noise from signal or to separate different signals.

The techniques for the efficient representation of signals are based on the orthogonality of the V and W spaces, which implies the non redundancy of the information in the wavelets representation. There exists some image formats based on the wavelets representation, such as ERMapper's ecw, which reaches very high compression rates. Some experiments of data compression with GRASS's wavelets modules are presented in [ZA02].

The application of operators in the W spaces can improve the computational efficiency, for example the matrix multiplication usually takes $N \times N$ operations, while in the wavelets representation it can be performed in $N \log N$ or even in N operations, depending on the operator analytical properties. This possbility has been exploited for the automatic extraction of surfaces for geomorphological studies, see [ZA02].

Signal filtering is based on the possibility of modifying the coefficient of a DWT and of reconstructing the signal without the features represented by the filtered out coefficients. The filtering is carried out in three steps:

1. signal decomposition;

2. modification of the coefficients at the different resolution levels;

3. signal recostruction.

The usual wavelets analysis and syinthesis is carried out in the spaces:

$$V_m = V_{m-1} \oplus W_{m-1} \tag{28}$$

Now the sequence of operations becomes:

$$V_m = V_{m-1} \oplus W_{m-1} \tag{29}$$

$$V_{m-1} \to \tilde{V}_{m-1} \tag{30}$$

$$W_{m-1} \to \tilde{W}_{m-1} \tag{31}$$

$$V_{m-1} = \tilde{V}_m \oplus \tilde{W}_{m-1} \tag{32}$$

The way the coefficients in the V_{m-1} and W_{m-1} spaces are modified into $\tilde{V}_m$ and $\tilde{W}_{m-1}$ depends on the filtering purpose, for example noise reduction, outliers detection or separation of digital surface models (DSM) from digital terrain model (DTM).

LIDAR surveyes deliver great performances in terms of speed and resolution but they are prone to outliers and noise. DTW can be used to lower the impact of these errors by filtering.

The outliers detection and elimination is performed by thresholding the coefficients in the "differences space" W; since in LIDAR surveys the outliers appears as point peaks the thresholding must carried out for the W space at the higher resolution. The coefficients rejection is done with "hard thresholding", following the so called "keep or kill" rule:

$$d_i^\star = \begin{cases} d_i & |d_i| \geq \lambda \\ 0 & otherwise \end{cases} \tag{33}$$

where λ is the threshold and d_i the generic ceofficient in the W space. The value of λ is chosen by calculating the distribution of the coefficients and setting the thresholding to the value corresponding

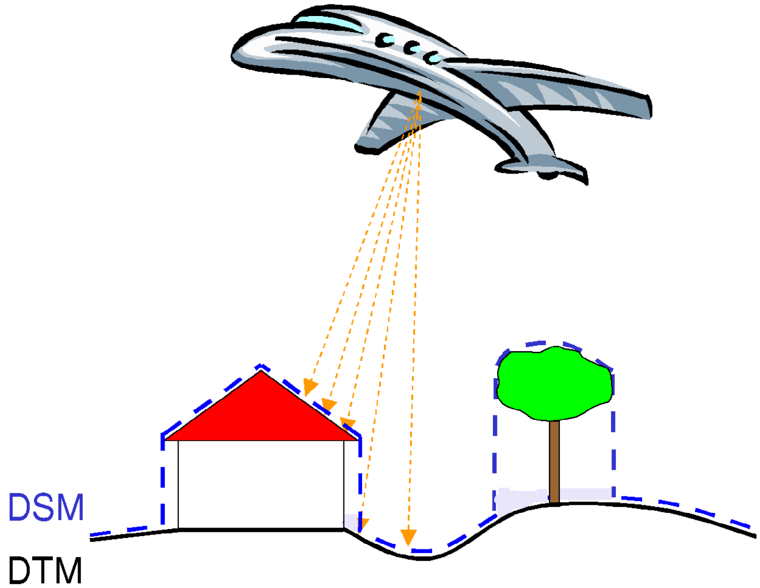

Figure 9: DSM and DTM from a LIDAR survey.

to an high probability of not being exceeded. In the application reported in [ZA02] the value is set to 97%.

Noise reduction is based on the orthogonality of DTW: white noise is transformed into white noise. It is possible to evaluate a *universal threshold* from the noise standard deviation σ, the number of saples n and the resolution level j:

$$\lambda_n = \sqrt{2\sigma^2 \log n} \tag{34}$$

The application of such a threshold is routinely used for image denoising, but can be used in a GIS for removing noise due to measurement errors or digitalization.

The separation of DSM and DTM is based on the possbility of selecting a feature by picking out the coefficients in the W spaces corresponding to the feature resolution and signal. While the first selection is made by choosing the proper W_m space, the signal characteristic is again filtered by thresholding the coefficient in the W_m space. In this way, selected features can be removed from the surface when it is reconstructed. It is possible to automatically detect the features entity, their position and, even detecting the features type, joining this techniques with other classification tools. Examples of application of this technique can be found in [ZA02], for both real and synthetic data.

Further applications are in progress for an urban area (Fig. 10).

An additional application of the GRASS wavelets modules is currently under investigation, examinating the possibility of using the transformed data in the V and W spaces as additional bands for surface classification. Different features can be combined for the classification:

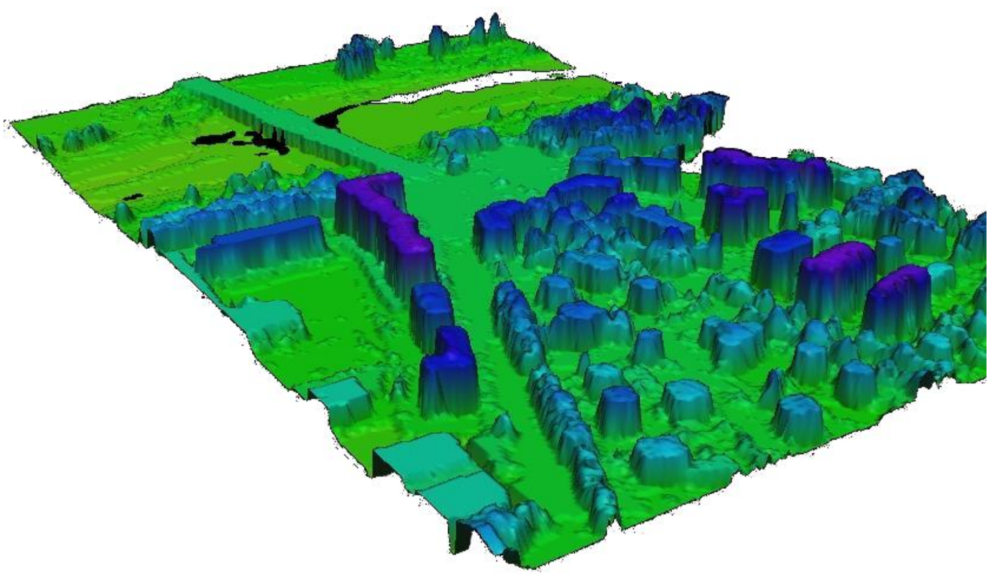

Figure 10: Surface of a part the city of Parma from LIDAR data.

- intensity of the signal on the different scales on the V_m spaces;
- variation of signal between resolution levels on the W_m spaces;
- texture on the v_m and W_m spaces.

8. Conclusions

The MRA approach is an efficient way to represent and elaborate data at different resolutions. This can be very useful in the contex of GIS, as the applications of paragraph 7. show, for the possibility of differenciate features at different resolution.

The complete separation of the V and W spaces allows the selection of the contribution of the features to the overall signal at the different scales: it is therefore possible to select features belonging to a known scale or with known signal pattern for their removal or emphatization.

By now, several aspects of the application of MRA in a GIS framework are under study:

- the determination of proper values for the thresholds;
- the choice of the wavelets bases with respect to the investigated phenomenon;
- the local filtering of signal in the W spaces.

A better understanding of these issues can lead to new and more efficient applications of data analysis in GIS, in particular of surfce and image analysis.

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